Contents

1. Microwave Principles 1-45

1.1. Introduction 1
1.2. Microwave Band 2
1.3. Millimeter Wave Band 3
1.4. Historical Background 4
1.5. Velocity of Waves Propagation in Media 7
1.6. Advantages of Microwave Line of Sight Communication 8
1.7.
1.8. Wave Propagation and Noise 10
1.9. Bending of Ray Path due to Atmospheric Refractivity Gradient 15
1.10. Estimation of Microwave Link Parameters 19
1.11. Application of Ray and Wave Theories in Microwave Links 30

2. Maxwell's Equations 46—58

2.1. Faraday's Law 46
2.2. Displacement Current 51
2.3. Maxewell's Second Equation 53

3. Transmission Lines 59-109

3.1. Introduction 59
3.2. Basic Transmission Line Equations 61
  3.2.1. Distortions in Transmission Lines 66
  3.2.2. Types of Transmission Lines 69
  3.2.3. Impedence Matching by use of Stubs 81
  3.2.4. Matching 83
  3.2.5. Concept of Transmission Line Converted to Waveguide 92
3.3. Electromagnetic Field Equations 95
3.4. Historical Background of Maxwell's Field Equations 97
3.5. Maxwell's Electromagnetic Field Equations for Electromagnetic Waves 101
3.6. Electromagnetic Plane Waves 102
3.7. Polarisation of Electromagnetic Waves 104

4. Waveguides 110—133

4.1. Introduction 110
4.2. Phase Velocity and Group Velocity in a Waveguide 111
4.3. Mathematical Analysis of Rectangular Waveguides 113
4.4. Transmission of TM Waves in Rectangular Waveguide 116

Electromagnetic wave: synchronized oscillation of electric & magnetic fields that propagate at the speed of light

1

Microwave Principles

chapter-7 (Frenzel) Art-7.2

can travel through vacuum of outer space

1.1 Introduction

Microwaves are radiowaves of very short wavelength. Basically radiowaves are electromagnetic waves the term radio being introduced from the concept of radiation of electromagnetic waves. Accordingly there should not be any lower or higher limits of the wavelengths. However, it is usual to confine the wavelength of radiowaves within certain limits to have electronics systems to generate, radiate and detect the radio waves, while infrared and optical waves are covered by systems called optoelectronic systems.

Table I

Radio wave bands (3kHz – 1GHz)

Frequency band	*Frequency range*	*Wavelength range*
Extra Low Frequency (ELF)	< 3 KHz	> 100,000 m
Very Low Frequency (VLF)	3 – 30 KHz	100,000 – 10,000 m
Low Frequency (LF)	30 – 300 KHz	10,000 – 1,000 m
Medium Frequency (MF)	300 – 3000 MHz	1,000 – 100 m
High Frequency (HF)	3 – 30 MHz	100 – 10 m
Very High Frequency (VHF)	30 – 300 MHz	10 – 1 m
Ultra High Frequency (UHF)	300 – 3000 MHz	100 – 10 cm
Super High Frequency (SHF) or Microwave	3 – 30 GHz	10 – 1 cm
Extra High Frequency (EHF) or Millimeterwave	30 – 300 GHz	10 – 1 mm

Microwaves have wavelengths which are limited to inclu wavelengths in the centimeter and millimeter wavelength region The entire frequency spectrum of radiowaves upto microwaves an millimeterwaves are, in fact, divided into a number of frequen bands as indicated in Table I, in which microwave and millimete wave bands cover the shortest wavelengths. The relation betwee the frequency f and wavelength λ being $f\lambda = c$, where c is velocity o propagation of the radiowave, which is equal to that of lightwav in free space, 3×10^{10} cm/sec $= 3 \times 10^{8}$ m/sec.

1.2 Microwave band (1GHz to 300GHz)

A glance at the various frequency ranges indicates that the UH band and the SHF range constitute the microwave frequency range However, instead of microwave the term 'microwaves' include both the microwave and millimeterwave band. Like wise, the term "millimeterwaves" include both millimeterwave and submillimeter wave [> 300 GHz].

It may be mentioned here that the unit of frequency which was originally called cycles per second is now called. Hertz, with the code Hz, after the name of Heinrich Hertz, who first demonstrated the existence of electromagnetic waves at microwave, ($\lambda = 66$ cm) 30 years after the theoretical prediction of electromagnetic waves by James Clerk Maxwell in the year 1858.

During world war II, the development of Microwave Radars in the frequency range 1–12 GHz prompted the defence engineers to subdivide the microwave range into four bands with the codes L, S, C, and X using the letters, in a way so that the bands may be kept secret through this code letters. These four bands cover the frequency ranges as shown in Table II.

Table II
Radar microwave bands

Frequency band	*Frequency range*
L	1 – 2 GHz
S	2 – 4 GHz
C	4 – 8 GHz
X	8 – 12 GHz

Subsequently, in recent years microwaves at frequencies higher than 12 GHz were generated and used for Radar and communication systems and those are also subdied into K, K and K bands as shown in Table III, which also includes the radar microwave bands, *L*, *S*, *C*, and *X*.

✓ Table III

Microwave bands

Frequency Band	*Frequency Range*
L	1 – 2 GHz
S	2 – 4 GHz
C	4 – 8 GHz
X	8 – 12 GHz
Ku	12 – 18 GHz
K	18 – 26.5 GHz
Ka	26.5 – 40 GHz

Radar & Communication System bands

1.3 Millimeterwave band

It may be noted that the *Ka* band covers a part of the millimeterwave band which starts from 30 GHz. However, the manufacturers of millimeter wave products extended the millimeter wave band down to 18 GHz as the same infrastructure for the production of millimeterwave products may be used to manufacture products down to 18 GHz. There is thus a overlap of microwave and millimeterwave bands over the frequency range 18 – 40 GHz covering *K* and *Ka* band of which 18 – 30 GHz is microwave and 30 – 40 GHz is millimeterwaves, as indicated in Table I. Manufacturers of Microwave products are now, in fact, developing products in the range 1 – 40 GHz, while Millimeterwave manufacturers are developing products in the range 18 – 300 GHz and even beyond 300 GHz. Thus there is a scope of choice between the Microwave and Millimeterwaves manufactures in the overlap bands *K* and *Ku*. Recently, some manufactures like Hewlett Packard are developing both Microwave and Millimeterwave products.

In view of the commercial interest millimeterwave manufacturers are now covering the band 18 – 300 GHz and even beyond 300 GHz upto about 400 GHz in the submillimeterwave region, and the millimeterwave band has been subdivided like that of microwave into a number of subbands as shown in Table IV, which also includes the *K* and *Ka* bands, included in Table III for the Microwave bands.

Table IV
Millimeterwave bands

Band	Frequency range
K	18 – 26.5 GHz
Ka	26.5 – 40 GHz
Q	33 – 50 GHz
U	40 – 60 GHz
V	50 – 75 GHz
E	60 – 90 GHz
W	75 – 110 GHz
F	90 – 140 GHz
D	110 – 170 GHz
G	140 – 220 GHz
H	170 – 260 GHz

It may be noted that there are overlaps of frequency bands between the *Ka* and *Q*, *Q* and *U*, *U* and *V*, *V* and *E*, *F* and *W*, *W* and *F*, *F* and *D*, and *D* and *G* bands.

In recent years, the entire spectrum of radio starting from ELF/VLF/LF/HF bands waves, may be integrated as shown in Fig. 1, covering Microwave and Millimeterwave bands, in which the frequency ranges in the UHF, SHF and EHF bands are indicated as 'Microwaves'.

1.4 Historical background

Historically microwave signal was first generated by Henrich Hertz in 1888 at 66 cm wavelength (454.5 MHz) while millimeter-wave signal was first generated by Sir J. C. Bose in 1895 at 5 mm wavelength (60 GHz). Subsequently, in 1890s, Bose also generated microwave signal at wavelengths upto 2.5 cm (12 GHz) wavelength. Besides these, Sir J. C. Bose also developed the world's first solid state point contact detector working at millimeterwave, infrared and optical wavelengths, using Galena (PbS) crystal as the detector material. Bose also developed world's first horn antenna and waveguide radiator for microwave and millimeter wave bands. In 1899 Sir J. C. Bose developed a highly sensitive iron-mercury detector in which a U-tube, made of glass, filled up with mercury, was used for a fine control of mercury contact pressure to optimise the sensitivity of the detector. Subsequently, in 1901 Marconi employed Sir J.C. Bose's technique of the highly sensitive iron-mercury detector.

Often the millimeterwaves, submillimeter waves ($f > 300$ GHz) and infrared waves are covered in an integrated manner under "infrared and millimeterwave" although the technologies for millimeterewaves as well as that of submillimeterwaves may be quite different from that required for infrared, where Optoelectronics Technology is prevalent. The reason for such an integration is that, some of the components are similar in design for millimeterwaves, submillimeterwaves and infrared, which are, in fact, borrowed from the systems used for optical waves, and called quasi-optical component. It may be mentioned here that many of the quasioptical components used for microwave and millimeterwave bands were first developed by Sir J. C. Bose in 1890s, to prove the quasi-optical properties of millimeterwave and microwave[2]. Sir J. C. Bose, in fact, developed a number of quasioptical components for microwave and millimeterwave bands, such as dielectric lens for focusing the wave to the detector, wire grid polariser for polarising the wave and also for analysing the polarised waves and curved cylindrical grating for accurate measurement of wavelength of microwave and millimeterwave. In addition to these, Sir J. C. Bose also worked for the first time on the response of living and nonliving meterials to microwave and millimeterwave bands, a subject which has now grown to a high level for the studies of microwave hazards to living beings, animals and vegetation and also for microwaves diathermy therapy of tumour and cancer. From the above history of Science about Sir J. C. Bose, who worked pioneeringly on a wide range of fields in Radio Science covering URSI commissions A. B. C. D. E. F. and K, it appears that Sir J.C. Bose may be called the father of Radio Science.

The difficulty of generating which, however, Bose generated by using his 'open resonator' coherent millimeterwaves and the limited distance of propagation of millimeterwaves upto line of sight distance apparently shifted the interest from millimeter waves to longer waves to wave length longer than a meter.

However, investigations of the millimeterwaves were continued and stimulated by scientific and millitary developments. For the purpose, feasibility of generating millimeterwaves of wavelength 1 cm (30 GHz) was demonstrated by Cleeton and Williams in studies of ammonia molecular resonances in the early 1930s. Subsequently in 1930 the observation by L. A. Hyland of the Naval Research Laboratory (NRL), about the reflection of radio signals from overflying aircrafts, stimulated the investigation of radar techniques for when a dielectric is placed in an electric field electric charges do not flow through the material)

1.5 Velocity of waves propagation in media

It may be mentioned here that although the velocity of propagation of radio waves and that of light wave are the same and equal to $c = 3 \times 10^{10}$ cm/sec when the medium is vacuum, in reality the propagation medium is usually not vacuum but may be the dielectric media like Teflon, Polysterene, Polyetheline, in which case the wave is slowed down, reducing the free space wavelength, λ_o, to dielectric wavelength λ_d, the two being related by the equation

(insulator

$$\lambda d = \frac{\lambda_o}{\sqrt{\varepsilon_r}} \quad \text{...(1)}$$

where ε_r = relative dielectric constant of the medium.

Even for propagation through air, we have to take into account the relative dielectric constant ε_r for the air medium, which is dependent on the humidity or watervapour density in the propagation path and temperature of the atmosphere. During rain or fog events, the effective relative dielectric constant may increase significantly to reduce the velocity of propagation of radio waves compared to that in clear air environment. The reduction of the velocity of propagation relative to that in free space causes some excess delay due to the atmospheric medium which may affect the estimation of target range by radar observations.

slow

vapor

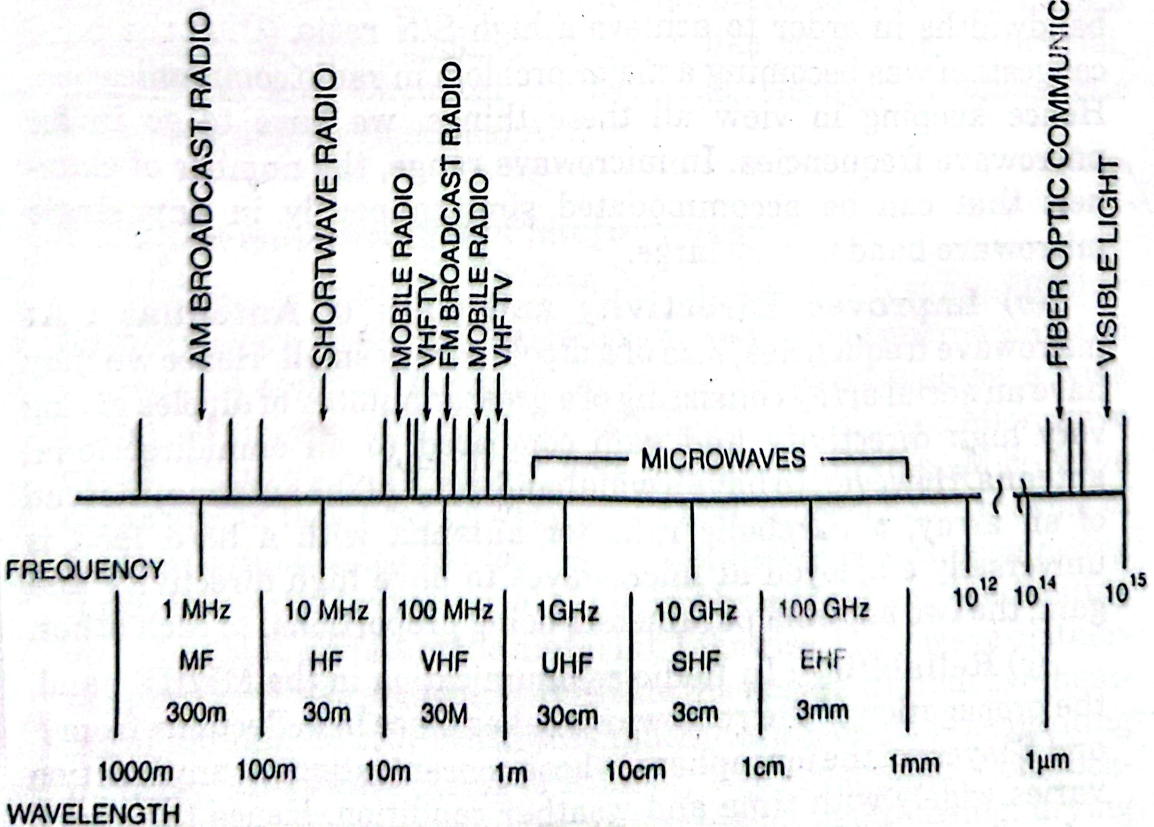

AM BROADCAST RADIO
SHORTWAVE RADIO
MOBILE RADIO
VHF TV
FM BROADCAST RADIO
MOBILE RADIO
VHF TV
MICROWAVES
FIBER OPTIC COMMUNICATIONS
VISIBLE LIGHT
FREQUENCY
1 MHz
10 MHz
100 MHz
1GHz
10 GHz
100 GHz
10^12
10^14
10^15
MF
HF
VHF
UHF
SHF
EHF
300m
30m
30M
30cm
3cm
3mm
1000m
100m
10m
1m
10cm
1cm
1mm
1μm
WAVELENGTH

Fig. 1:1.

1.6 Advantages of microwave line of sight communication

The microwave systems for radar and communication are becoming more and more important in Navigation, ground based line of sight communication, satellite communication, and other fields like remote control and remote sensing.

The various reasons pertaining to the wide use of microwaves are underlined as below :

(*a*) Broader band width

(*b*) Improved directivity and higher gain of antennas.

(*c*) Higher Reliability and

(*d*) Lower Transmitter Power requirements

(*a*) **Band Width :** Telegraph, speech, music and video signals may be transmitted directly through wireline or cable as a baseband signal covering baseband width for each type of signals. We know that a telegraph channel requires a bandwidth at 120 Hz, the speech requires a bandwidth of 2–3 kHz, a music channel requires a bandwidth of the order 10–20 kHz, each one as the base bandwidth, whereas the bandwidth required by TV channel is very high, being 7 MHz, to include the video bandwidth of 5 MHz together with the bandwidth of the frequency modulated audio signal. Now the entire frequency range upto 1000 MHz or 1 GHz is crowded with various radio channels, telegraph channels, TV channels etc. Also with the advent of frequency modulation we have to go in for higher bandwidths in order to achieve a high S/N ratio. Thus the band congestion was becoming a major problem in radio communication. Hence keeping in view all these things, we have to go in for microwave frequencies. In microwave range, the number of channels that can be accommodated simultaneously in any single microwave band is very large. as unidirectional (antenna used)

(*b*) **Improved Directivity and Gain of Antennas :** At microwave frequencies, size of a dipole is very small. Hence we may have an aerial array consisting of a greater number of dipoles giving very high directivity and gain compared to an omnidirectional antenna. However, to have a wide bandwidth of the antenna, instead of an array, a parabolic reflector antenna with a horn feed is universally employed at microwaves to have high directivity and gain, the two antenna parameters being proportional to each other.

(*c*) **Reliability :** In Radio communication in the MF/HF band, the propagation of the radiowaves takes place by reflections from *F* and *E* layers of the ionosphere, whose concentrations at any location varies widely with time and weather condition. Hence the signal received varies widely with time and weather. Hence the energy received by the receiver is not of uniform strength but there are great time variations in the strength of received signal, though the transmitted power may be constant, thus giving rise to Fading Effect. But at microwave frequencies, there is less fading since the propagation of microwaves from transmitter to receiver takes place by line of sight propagation through the neutral atmosphere. As the frequency increases, the reception becomes clearer in microwave ranges with still less fading, as the fading at microwaves occurs only in the presence of atmospheric turbulence created by winds in the atmosphere while the ionosphere is transparent to microwaves allowing stable propagation between satellite and earth based microwave communication links.

(*d*) **Power Requirements :** At microwaves, the power requirements of the transmitter become very small as compared to that at MF/HF, due to the high gain of the antennas at microwaves. The basic configuration of the transmitter as well as the receiver at microwaves is similar to that in MF/HF bands but the active and passive components used are different. The only disadvantage of microwaves being the need for a number of repeater stations, as a microwave line of sight link over the ground cannot have a range greater than about 40–60 km due to curvature of the earth's surface. For earth-space communication with satellites, the range is about 3 orders of magnitude higher, being about 36,500 km for geostationary satellites with circular equatorial orbit. Microwaves have a widerange of other application areas like consumer, Industrial, Biomedical and Chemical applications, electron paramagnetic resonance and Navigational Aid.

1.7 Other Applications of Microwaves

Microwave energy has, in fact, a heating effect just as any other form of energy. Because of very short wavelength of microwaves and consequent compact circuitry, this heating effect has got a wide variety of non-military or civilian applications including Microwave ovens, Microwave diathermy and Microwave drying. Microwave ovens are, in fact, used for very quick and uniform home cooking of food materials, since the food is cooked by the waves penetrating inside the entire food material to all depths without any overheating on the outside surface of the material. Likewise, Microwave diathermy machines also produce heat inside the muscle without overheating of the skin. Microwave drying machines are used in the printing industry, Microwave curing of Rubber, Tire and Asphalt from socked water, in Microwave material processing like thawing, drying and heating and also in Agricultural industries for vacuum drying of sliced parsely root, cotton seed, microwave treatment of seed, drying of field corns and heating of corn fields, warming of plants, for use as insectisides in grain, Tobacos and Pecan Weevil, and for pasteurisation of milks. Even for Forest products microwaves are applied for drying of Veneer and Bending of Wood.

1.8 Wave Propagation and Noise

Line of sight propagation at microwave and millimeterwave bands, in atmospheric medium, usually produces an excess attenuation over and above that expected for propagation through free space in vacuum. The atmospheric water vapour, liquid water content and temperature are, in fact, the main determining factors controlling the excess attenuation in the atmosphere. Typical values of the atmospheric excess attenuation in the microwave and millimeter-wave bands is shown in Fig. 1.2 (*a*) which also shows the contributions of rain and fog on the excess attenuation. The figure shows that in microwave line of sight links, the atmospheric excess attenuation in clear atmosphere is as low as 0.01 dB/km for frequencies upto 10 GHz. However, for the excess rain attenuation, it may be as high as 10 dB/km at 10 GHz for a rain rate of 150 mm/hr. The

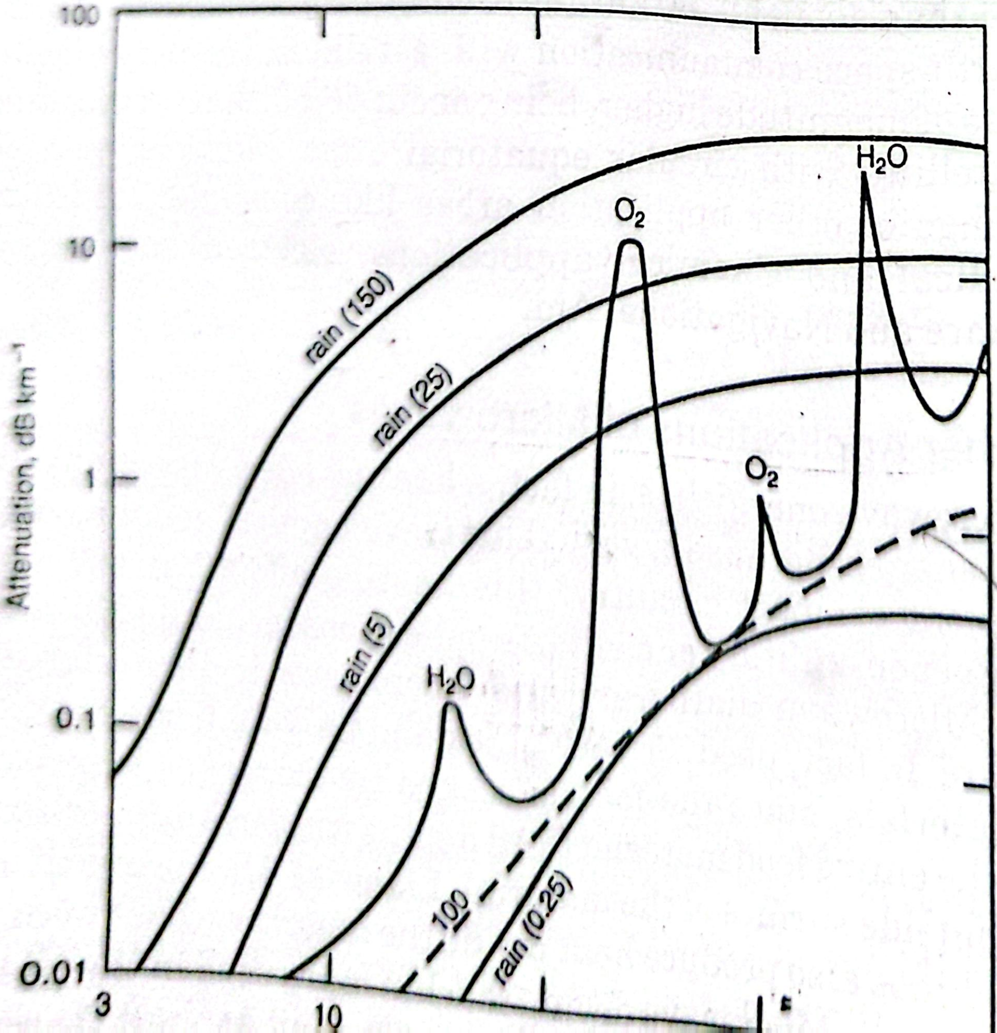
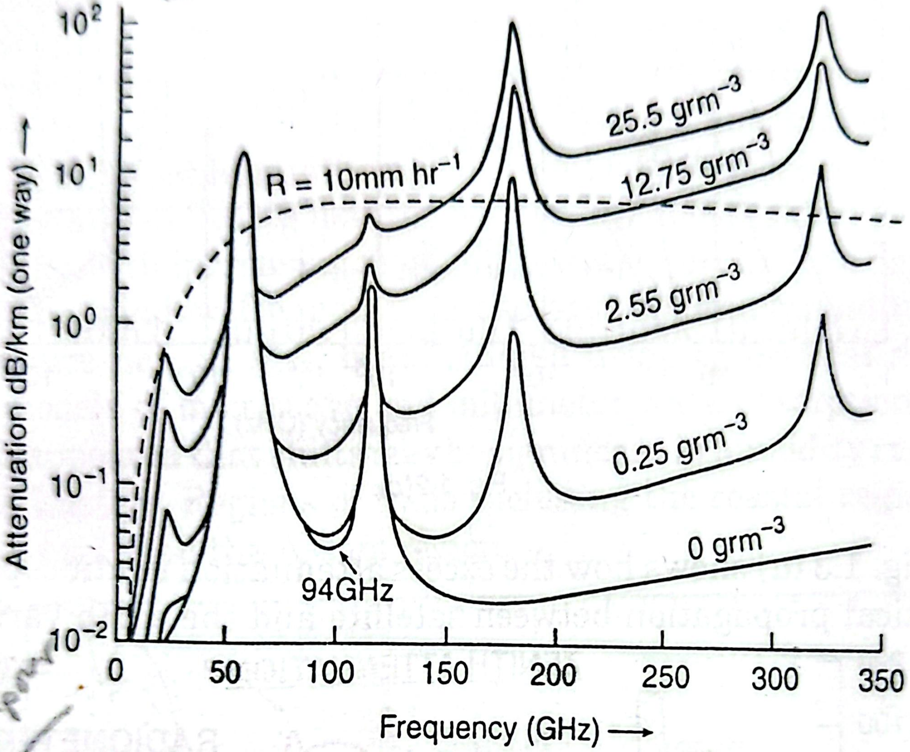


figure also shows that in clear atmosphere without rain there are a few absorption lines of oxygen and water vapour molecules of the atmosphere, being around 22 and 183 GHz for water vapour, and at 60 and 120 GHz for oxygen. The figure also shows that the excess attenuation due to fog is very low and starts only beyond about 20 GHz, the amount being comparable to rain attenuation for a rain rate of only 0.25 mm/hr. In clear air the attenuation increases with an increase of water vapour content as shown in Fig. 1.2 (*b*), which



For water vapour

Fig. 1.2(*b*).

also shows that the excess attenuation above 100 GHz due to rain rate of 10 mm/hr is about 10 dB/km which is around the clear air attenuation with water vapour density of 25.5 gm m^{-3}.

For satellite communication, where the ray path is slanting, the attenuation is less, particularly, at frequencies between the absorption lines. But the amount increases rapidly with water vapour content or the humidity of the atmosphere. For frequencies above 300 GHz in the submillimeterwave, infrared and optical regions, the atmospheric excess attenuations in clear weather and in fog are shown in Fig. 1.2 (*c*).

For clear weather the excess attenuation at submillimeterwave is more than 100 dB/km, while at infrared and optical bands, it is very low, in the range 0.1–1.0 dB/km as shown in Fig. 1.2 (*c*), which also shows that for fogy weather with a water vapour density of 0.1 g/m^3 the excess attenuation becomes as high as 100 dB/km in the infrared and optical regions.

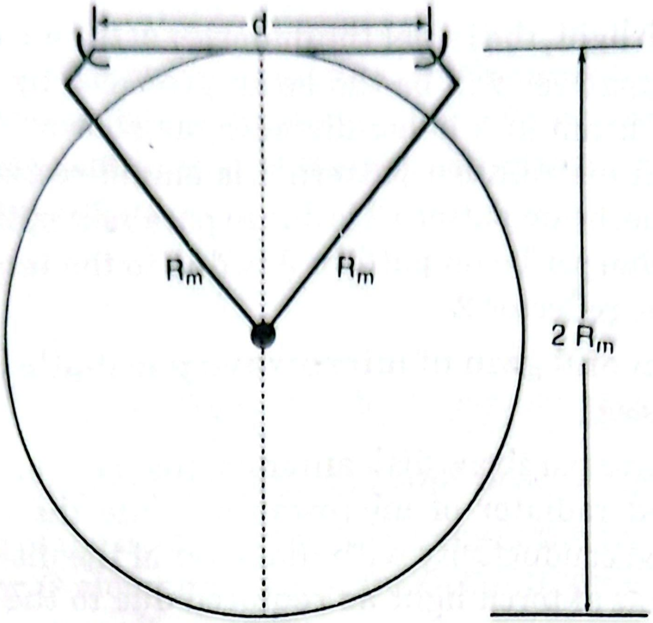
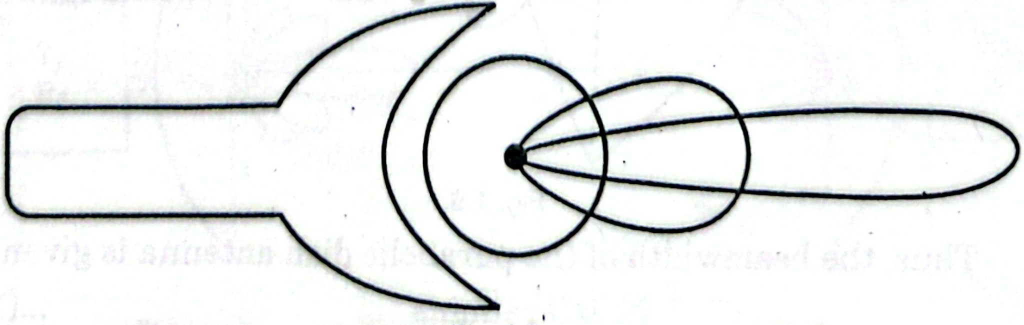


Fig. 1.7.(c)

1.10. Estimation of Microwave Link Parameters

For a microwave line of sight link, after estimating the maximum line of sight distance from equation (1.3) the requirements of transmitter power and receiver sensitivity can be estimated assuming the loss of radiated power with distance, following the inverse square law, assuming if the atmospheric attenuation to be is negligibly small at the microwave frequency chosen for the link. Also, the transmitter power has to be estimated by taking into account the gains of the radiating and receiving antennas. The gain of an antenna, in fact, arises from the focussing effect of the parabolic dish antenna with horn fed used in microwave links. The situation, is, in fact, similar to the increase of the intensity of the light beam produced by torch lights due to the focussing effect of the parabolic optical reflector of the torch, with a torch bulb located at the focal point, of the reflector, to convert the omni-direction radiation pattern of the torch bulb, into a narrow light beam of much higher intensity. The intensity of the light beam due to the focussing effect of the reflector depends on the diameter of the parabolic optical reflector of the torch. At microwaves, the torch bulb is replaced by a horn feed radiator located at the local point of the parabolic metallic dish antenna acting as a reflector for microwaves.



In the torch light, the larger the diameter of the optical parabolic reflector the narrower will be the beam produced by focussing, to get a brighter beam at a given distance, as shown in Fig. 1.8, in which the light distribution pattern 1 is omnidirectional, due the torch bulb alone, beam pattern 2 is due to parabolic optical reflector, D_1 while the sharper beam pattern 3 is due to the larger diameter of the parabolic reflector 2.

1.10.1 Pattern and gain of microwave parabolic dish antennas

In microwave parabolic dish antenna, the torch bulb is replaced by a horn feed radiator of microwaves, while the dish is made metallic of good conductivity with diameter of the dish made much larger than that of torch light as required due to the much longer wavelength of microwaves compared to that for torch light. The structure of a microwave dish antenna is shown in Fig. 1.9. In this the microwave feed horn located at the focal point of the dish, is not however, omnidirectional but the beam of the horn feed is broad enough to cover the full area of the parabolic dish as shown in Fig. 1.9 with no power being lost by radiating millimeterwave microwaves in the direction away from the dish area. The width of the microwave beam can be estimated by using the same equation as used for estimating the beamwidth for the torch light beam, the only difference between the light and microwave dish being the wavelength.

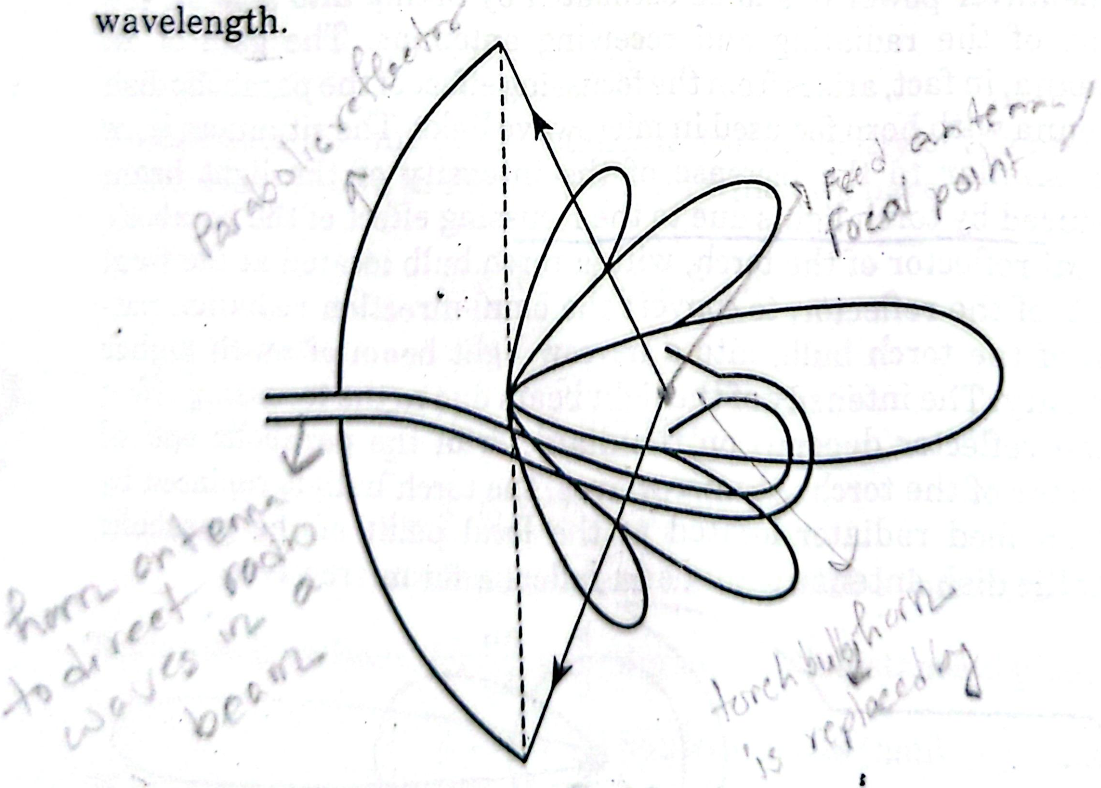


Fig. 1.9.

Thus, the beamwidth of the parabolic dish antenna is given b

$$\theta = 1.22\ \lambda/D \text{ radians} \qquad \ldots(1.$$

$= 1.22 \times 180 \times \lambda/D$ degree

Power flux density provides the measure of energy that flows through a unit area each second;

$$= 70\ \lambda/D \text{ degrees} \qquad \ldots(1.7)$$

and it can be shown that the gain G of the antenna, which determines the power flux of the beam at the beam centre at a given distance compared to that of the omnidirectional pattern without the dish, is given by

$$G = \frac{4\pi A}{\lambda^2}$$

→ ছিদ্র

where, $A = \frac{\pi D^2}{4}$ is the front mouth or aperture area of the dish.

1.10.2. Effective radiated power and power received

The use of the dish effectively increases the transmitter power by the factor G along the axis of the beam to produce an effective transmitter power, P, given by

$P = P_T G_T$, where P_T is the real power of the microwave transmitter. P is called the Effective Isotropic Radiated Power (EIRP). We can then calculate the power flux along the beam centre reaching the receiving end by using the inverse square law of the fall of power with distance, as in optics. Thus the power flux, P_f, at the receiving antenna aperture at a distance from the transmitting dish antenna is given by $P_f = \frac{P_T G_T}{4\pi d^2}$. If the diameter of the receiving dish antenna is also the same as that of the transmitting dish antenna, each being of aperture area being given by a $A = \frac{\pi D^2}{4}$, then the power, P_r, picked up by the receiving dish antenna is given by

$$P_r = P_f \times A = \frac{P_T G_T}{4\pi d^2} \times A = \frac{P_T G_T}{4\pi d^2} \frac{\pi D^2}{4}$$

Fig. 1.10 shows the configuration of typical microwave link with parabolic dish antennas.

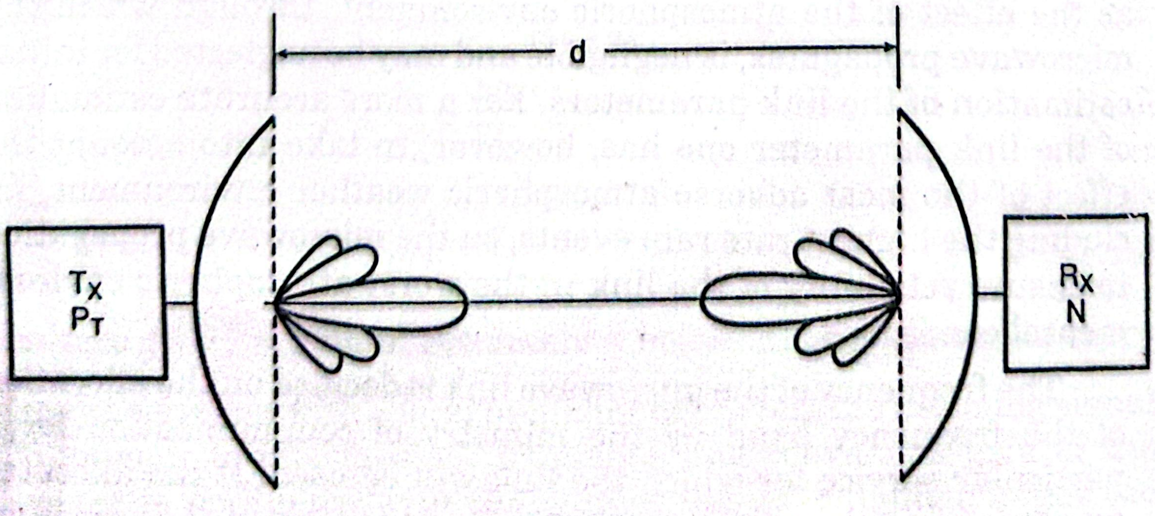


Fig. 1.10.

It can be shown from the reciprocity theorem that the gain of a parabolic dish antenna is related to its area A by the equation

$$G = \frac{4\pi A}{\lambda^2} = \frac{4\pi}{\lambda^2}\,\frac{\pi D^2}{4} = \frac{\pi^2 D^2}{\lambda^2} \qquad \text{...(1.9)}$$

from which we have

$$P_r = \frac{P_T G_T}{4\pi d^2} \times A \quad P_r = \frac{P_T}{4\pi d^2}\,\frac{\pi^2 D^2}{\lambda^2}\,\frac{\pi D^2}{4}$$

$$= \frac{P_T}{4d^2}\,\frac{\pi^2 D^4}{4\lambda^2}$$

$$= \frac{P_T \pi^2 D^4}{16\, d^2 \lambda^2} \qquad \text{...(1.10)}$$

$$\cong \frac{10\, P_T D^4}{16\, d^2 \lambda^2}, \text{ as } \pi^2 \cong 10$$

or

$$P_r = \frac{P_T D^4}{1.6\, d^2 \lambda^2} \qquad \text{...(1.11)}$$

If the diameters of the Transmitting and receiving dishes are different, D_1 and D_2 respectively, than we have

$$P_r = \frac{P_T D_1^2 D_2^2}{1.6\, d^2 \lambda^2} \qquad \text{...(1.12)}$$

Now the wavelength λ is related to frequency f through the relation

$$f\lambda = c \qquad \text{...(1.13)}$$

or

$$\lambda = c/f \qquad \text{...(1.13)}$$

Where c is the velocity of propagation of the microwaves, that may be assumed to be equal to that of light propagating in free space, as the effect of the atmospheric environment, through which the microwave propagates, is negligible and may be neglected for initial estimation of the link parameters. For a more accurate estimation of the link parameter one has, however, to take into account the effect of the most adverse atmospheric weather environment, including the highest rate rain events, on the microwave propagation to ensure reliability of the link in the worst atmospheric environmental conditions.

The frequency of the microwave link is decided on the allocation of the frequency band by the ministry of communication for a particular service for which the link will be used. If the allocated frequency be f then the equation for power received P_r is given by equation (1.12) as

SNR = $\frac{\text{average signal power}}{\text{average noise power}}$

$$P_r = \frac{P_T D_1^2 D_2^2}{1,6\, d^2 \lambda^2} = \frac{P_T D_1^2 D_2^2}{1.6\, d^2 (c/f)^2}$$

$$= \frac{P_T D_1^2 D_2^2 f^2}{1.6\, d^2 c^2} \quad \checkmark \qquad \text{...(1.14)}$$

This received power P_r available at the output terminal of the receiving antenna should be sufficiently larger than the inherent noise level of the receiver, so that the signal to noise ratio, (S/N) before demodulation produce the signal to noise ratio, $(S/N)_D$, after demodulation, which may be high enough to produce an acceptable signal to noise ratio of the demodulated signal from the microwave receiver, to be sent to the customer-user of the link.

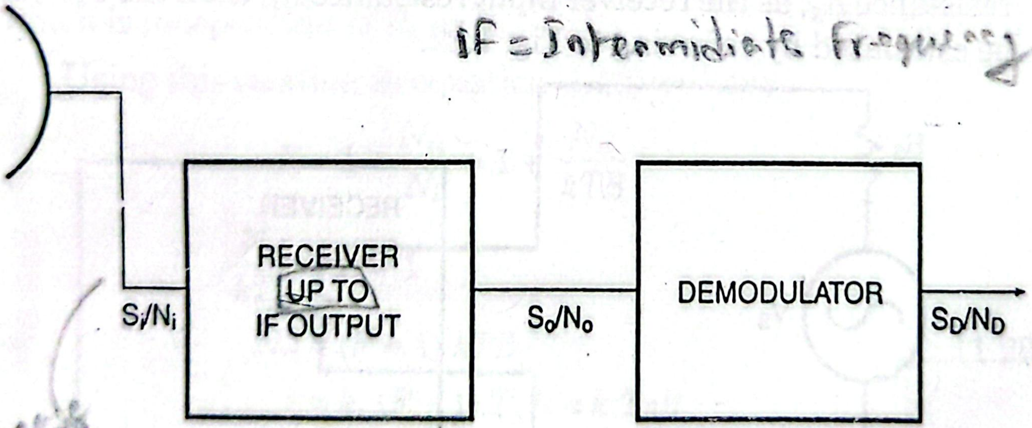
1.10.3. Receiver noise level and system noise

Limitation of signal to noise ratio due to the noise level of the receiver is to be estimated in designing a link. The noise figure of the receiver F is defined by the equation

$$F = \frac{(S/N)_i}{(S/N)_o} \qquad \text{...(1.15)}$$

where $(S/N)_i$ is the signal to noise ratio at the receiver input and $(S/N)_o$ is the signal to noise ratio of the receiver output before demodulation, Equation (1.15) can be rewritten as

IF = Intermediate frequency



Inserting

Low noise RF Amplifier → $N = N_p + gN_i$

Fig. 1.11.

$$F = \frac{S_i}{S_o} \times \frac{N_o}{N_i}$$

$$= \frac{N_o}{gN_i} \qquad \text{...(1.16)}$$

where $g = \frac{S_o}{S_i}$ = gain of the receiver before demodulation. As there may be a mixer with a local oscillator to convert the incoming signal frequency to intermediate frequency, the conversion gain of the mixer is multiplied with the gain of the 1F amplifier to get the

Thermal noise: Random motion of electrons in a wire which creates an extra signal not originally sent by the TR

receiver gain before demodulation. In case, where as a low noise R amplifier is used before the mixer, the gain of the RF amplifier i also included in the receiver gain.

Now $N = g\,N_i + N_r$ ...(1.17)

as, N is the sum of noise power produced in the receiver N_r plus th noise power at the input of the receiver amplified g times by th receiver. Using the relation (1.16) in the equation (1.17) we have

$$F = \frac{gN_i + N_r}{gN_i} = 1 + \frac{N_r}{gN_i}$$

$$= 1 + \frac{N_r}{g} \times \frac{1}{N_i}$$

$$= 1 + \frac{N_{ri}}{N_i} \qquad ...(1.1$$

where $N_{ri} = \frac{N_r}{g}$ is the receiver noise power referred to its input. Th noise power N_i at the receiver input is normally assumed to be th thermal noise power produced by the resistance of the receive matched to the antenna radiation resistance. This is valid for groun based line of sight links at microwaves. If the receiver input resistance R_i is matched to a signal generator having the same outpu resistance R_g, as the receiver input resistance R_i, then the S_i/N_i may be estimated by referring to Fig. 1.12.

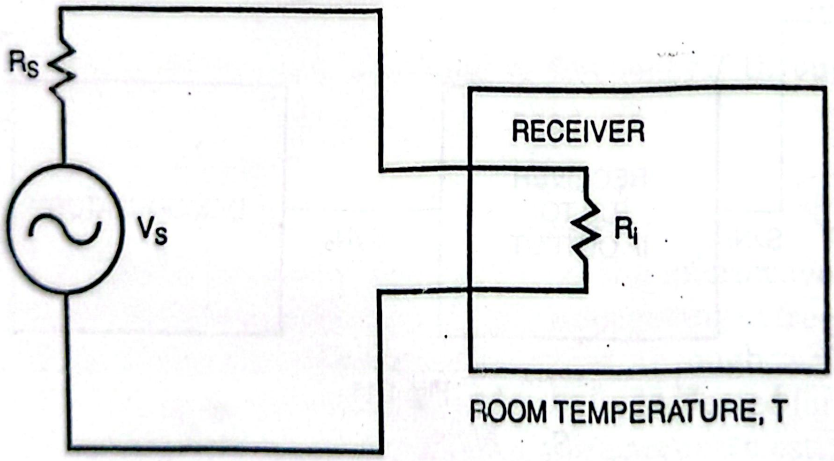


Fig. 1.12.

$$S_i = i^2 R_i$$

where i is the microwaves current flowing between the generato and the receiver

or $$S_i = \left[\frac{V_s}{R_s + R_i}\right]^2 R_i$$

Radiation resistance: Feed point resistance that is produced by the radiation of the electromagnetic wave from antenna, and heat up the antenna.

where V_s is the microwave voltage generated at the signal source of the generator.

or
$$S_i = \frac{V_s^2 R_i}{[R_s + R_i]^2}$$

maximum

For optimum power transfer of noise power from the signal source to the receiver input we make $R_i = R_s$, when we have

$$S_i = \frac{V_s^2}{(2R_i)^2} R_i = \frac{V_s^2}{4 R_i}$$

The thermal noise voltage produced by R_s at the environment of room temperature T, can be shown to be given by $V_n = \sqrt{4kTBR_s}$.

where $k = 1.38 \times 10^{-23}$ is the Boltzman's constant and B is the receiver bandwidth.

$\therefore$ Noise power at the receiver input N_i is given by

$$N_i = \frac{V_n^2}{(R_s + R_i)^2} \times R_i$$

$$= \frac{4kTBR_s}{4 R_s^2} \times R_s \text{ under matched condition with } R_i = R_s$$

or
$$N_i = kTB, \qquad \text{...(1.19)}$$

which is independent of R_s or R_i.

Using this relation in equation (1.18) we have

$$F = 1 + \frac{N_{ri}}{N_i} = 1 + \frac{N_{ri}}{kTB}$$

$$\therefore \qquad \frac{N_{ri}}{kTB} = F - 1$$

or
$$N_{ri} = (F - 1)\, kTB \qquad \text{...(1.20)}$$

$$= k.\, (F - 1)\, T.\, B = k\, T_R B$$

where $(F - 1)\, T = T_R =$ noise temperature of the receiver.

For microwave line of sight link the total noise power referred to the input of the receiver is given by

$$N = N_i + N_{ri} = kTB + (F - 1)\, kTB = FkTB \qquad \text{...(1.21)}$$

by combining equations (1.19) and (1.20).

For microwave link, as the dish antenna is located on the ground and antenna beam is directed horizontally over the ground towards the receiving dish antenna also installed at ground, the antenna radiation resistance will have an effective temperature equal to the temperature, T, near the ground environment for which $N_i = kTB$

and equation (1.21) is thus valid. However, in case the antenna beam is directed upwards, as in satellite communication links connecting the satellite with the ground station by a microwave link, N_i may be much less than kTB, as the antenna is looking towards the sky where the sky temperature is much lower than that of the environment near the ground as the contribution of the sky noise is the thermal noise of atmospheric water vapour in the sky at a very low temperature together with some amount of galactic noise. Both of these noise sources are much worker than the noise from the ground level atmospheric environment due to water vapour and that from the earth's surface, which are around the room temperature T, in case of line of sight ground based microwave links. ✓

1.10.4. Receiving system noise temperature for satellite link

For estimation of the lower noise level picked up by antenna with its beam pointed upwards, as in Satellite Communication we make use of the antenna temperature T_a which is the equivalent temperature, to which the antenna radiation resistance or the terminal resistance, must be raised to account for the noise picked up by antenna from the sky. In that case $N_i = k\,T_a B$ instead of kTB and thus the total noise power referred to the input of the receiver, is given by

$$
\begin{aligned}
N = N_i + N_{ri} &= k\,T_a\,B + k\,(F-1)\,TB \\
&= k\,T_a\,B + k\,T_R B = k\,(T_a + T_R)\,B \\
&= k\,T_{sys}\,B \qquad \text{...(1.22)}
\end{aligned}
$$

where $T_a + T_R = T_{sys}$ is the equivalent system noise temperature of the satellite receiving system.

1.10.5 Complete link design equations

Fig. 13 shows a microwave link set up for which the design equations are summarised as follows :

Antenna beam widths $\theta_1 = 70\dfrac{\lambda}{D_1}$, and $\theta_2 = 70\,\dfrac{\lambda}{D_2}$

Antenna gains $G_1 = \dfrac{\Pi^2 D^2}{4}$ and $G_2 = \dfrac{\Pi^2 D^2}{4}$.

Power received at the receiving end for the down link operating at f_1.

$$P_{r_1} = \frac{P_{T1}\,D_1^2\,D_2^2}{1.6\,d^2\,\lambda^2} \qquad \frac{P_{T1}\,D_1^2\,D_2^2\,f_1^2}{1.6\,d^2 c} \qquad \text{...(1.23)}$$

For the uplink at f_2 the satellite receiving system will have the received power, P_{r_2}, given by

$$P_{r_2} = \frac{P_{T2}\, D_1^2\, D_2^2}{1.6\, d^2\, \lambda^2} = \frac{P_{T_2}\, D_1^2\, D_2^2\, f_2}{1.6\, d^2 c} \qquad \text{...(1.24)}$$

For two way simultaneous links, the two frequencies are required to be used for forward or uplink and backward or down link, so that the high transmitter power at any one end may not leak into the highly sensitive receiving system at the same end due to insufficient isolation between the transmitter and receiver. The forward and reverse transmission frequencies are, in fact, separated by a minimum amount required to avoid the problem of high power transmission leaking into the receiver of the same end. The difference of the frequencies f_1 and f_2 can not, in fact, be made large primarily for the reason that the same dish antenna with its feed used for the up/down link cannot be developed to cover a widely separated up/down link frequencies f_2 and f_1. For this reason, the frequency difference between f_1 and f_2 for the downlink and uplink, respectively, is kept within about 3 GHz.

The signal power S, before demodulation at the receiver input is often called the carrier power C and it must be, at least, 20 dB above the total receiver noise power both referred to the receiver input, so that after demodulation C/N can reach a value of the order of $(S/N)_D = 40 - 50$ dB, acceptable to the consumers. The increase in the $(S/N)_D$ ratio after demodulation of C/N is due to the nonlinear processing mechanism of the demodulator and the ratio $p_r = \dfrac{(S/N)_D}{C/N}$ is called the processing gain of the demodulator. The value of p_r depends on the type of modulation and the modulation index.

1.10.6. Worked out problem on link design

Example 1.1. *A microwave line of sight link at 10 GHz is to be designed for communication over a distance of 10 km. If the diameter of the dishes at the two ends of the link are the same and equal to 3m, calculate the transmitter power required when the receiver noise figure at the receiving end is 10 dB, receiver bandwidth is 1 GHz and the C / N ratio is required to be 40 dB.*

Ans. From equation for power received P_{ri} we have

$$P_{ri} = \frac{P_T\, D_1^2\, D_2^2\, f^2}{1.6\, d^2\, c^2} = \frac{P_T\, D^4\, f^2}{1.6\, d^2\, c^2}$$

where $D_1 = D_2 = D = 3\text{m}$

$f = 10 \text{ GHz} = 10 \times 10^9 \text{ Hz}$

$$d = 30 \text{ km} = 30 \times 10^3 \text{ m}$$

$$c = 3 \times 10^8 \text{ m/sec}$$

$$\therefore \quad P_{ri} = \frac{P_T \times 3^4 \times [10^2 \times 10^9]^2}{1.6 \times (30 \times 10^3)^2 \times [3 \times 10^8]^2}$$

$$= \frac{P_T \times 3^4 \times 10^{20}}{1.6 \times 3^2 \times 10^8 \times 3^2 \times 10^{16}} = \frac{P_T\, 10^{-4}}{1.6} W$$

Total noise power, N, at the receiver input is given by

$$N = F\, kTB$$

$$10 = 4 \times 1.38 \times 10^{-23} \times 300 \times 10^9$$

where $T = 273 + 27 = 300$ for the environmental temperature 27°C.

$$= 4 \times 1.38 \times 300 \times 10^{-23} \times 10^9\ W$$

$$\cong 10 \times 4 \times 10^{-21} \times 10^9\ W$$

$$C = 40 \times 10^{-12 \text{ watt}}$$

where C is the carrier power.

As, the carrier power, C, is to be 40 dB above N we should hav

$$C = 40 \times 10^{-12} \times 10^4 = 40 \times 10^{-8} \text{ Watt}$$

Assuming, $P_{ri} = C$, we have

$$\frac{P_T}{1.6} \times 10^{-4} = 40 \times 10^{-8}$$

$$\therefore \quad P_T = 40 \times 1.6 \times 10^{-8} \times 10^4 \text{ W}$$

$$= 64 \times 10^{-4} \text{ W} = 6.4 \times 10^{-3} \text{ W} = 6.4 \text{ mW}.$$

Example 1.2. *A satellite communication link for earth statio has got a parabolic dish antenna of diameter 10 m fed from microwave transmitter at 6 GHz having a power of 3 kW. If t overhead geostationary satellite has a microwave dish antenna diameter 2 m, calculate the C/N ratio at the satellite receivi terminal for the up link having a noise temperature 300 K and bandwidth of 500 MHz.*

Solution. We have for the uplink at 6 GHz

$$P_{ri} = \frac{P_T D_1^2 D_2^2 f^2}{1.6 \times d^2 c^2} = \frac{3 \times 10^3 \times 10^2 \times 2^2 \times (6 \times 10^9)^2}{1.6 \times (36500 \times 10^3)^2 (3 \times 10^8)^2}$$

$$= \frac{3 \times 2^2 \times 6^2 \times 10^{23}}{1.6 \times (3.65)^2 \times 10^{14} \times 3^2 \times 10^{16}}$$

$$= \frac{12 \times 36 \times 10^{23}}{1.6 \times 13.32 \times 9 \times 10^{30}} = \frac{432 \times 10^{23}}{191.81 \times 10^{30}} \text{ W}$$

$$= 2.252 \times 10^{-7} \text{ W} = 0.2252 \ \mu \text{ W}$$

$$N = k\, T_{sys}\, B = 1.38 \times 10^{-23} \times 300 \times 500 \times 10^{6}$$

$$\cong 4 \times 10^{-21} \times 5 \times 10^{8} = 20 \times 10^{13} \text{ W}$$

$$= 2 \times 10 - 12 \text{ W}$$

$$\therefore \quad \frac{C}{N} = \frac{P_{ri}}{N} = \frac{2.252 \times 10^{-7}}{2 \times 10 - 12} \cong 1 \times 10^{5} = 50 \text{ dB}$$

Example 1.3. *If the transmitter power of the satellite terminal is 200 W, calculate the C/N ratio of the down link at 4 GHz, at the earth station, assuming the noise temperature of the receiving system of the earth station to be 50 K with the receiver band width is 500 MHz.*

Solution. We have for the down link at 4 GHz

$$P_{ri} = \frac{200 \times 2^2 \times 10^2 \times (4 \times 10^9)^2}{1.6 \times (36500 \times 10^3)^2 \times (3 \times 10^8)^2} \text{ W}$$

$$= \frac{2 \times 2^2 \times 10^4 \times 4^2 \times 10^{18}}{1.6\, (3.65 \times 10^7)^2 \times 3^2 \times 10^{16}} \text{ W}$$

$$= \frac{128 \times 10^{22}}{1.6 \times 13.32 \times 10^{14} \times 9 \times 10^{16}} \text{ W}$$

$$= \frac{128 \times 10^{22}}{191.81 \times 10^{30}} \text{ W} = 0.6673 \times 10^{-8} \text{ W}$$

$$N = k\, T_{sys}\, B$$

$$= 1.38 \times 10^{-23} \times 50 \times 500 \times 10^{6}$$

$$= 1.38 \times 25 \times 10^{-23} \times 10^{9} \text{ W}$$

$$= 1.38 \times 25 \times 10^{-14} \text{ W}$$

$$\frac{C}{N} = \frac{P_{ri}}{N} = \frac{0.6673 \times 10^{-8}}{1.38 \times 25 \times 10^{-14}}$$

$$= \frac{6.673 \times 10^{-9}}{34.5 \times 10^{-14}}$$

$$= \frac{6.673 \times 10^{-9}}{3.45 \times 10^{13}}$$

$$= 1.9342 \times 10^{4}$$

$$= 40 + 2.865 \text{ dB} \cong 43 \text{ dB}$$

10log( ) dB

2

Maxwell's Equatio

In this chapter, electric and magnetic fields that change wi time are discussed and a number of new relations and concepts a introduced. Some of these concepts are :

1. *Faraday's Law*. This gives the e.m.f. induced in a clos circuit due to a change of magnetic flux linking it.

2. A relation giving the e.m.f. induced in a conductor moving a magnetic field.

3. Maxwell's displacement current which is an extension of t concept of current to include charge free space.

4. Extension of boundary relations to include time var situations.

2.1. Faraday's Law

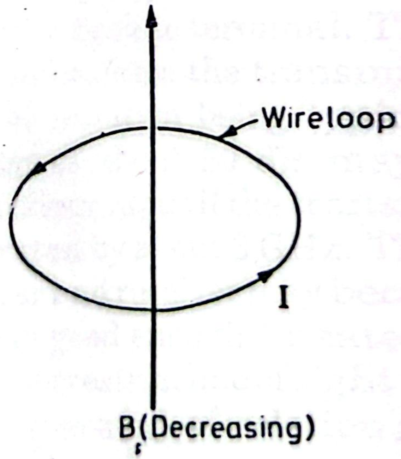


Fig. 2.1. Relation between decreasing flux density *B*, band induced current, I

We know that a current carrying conductor produces a magnetic field. In 1831, Faraday found that a magnetic field can produce a current in a closed circuit provided that the magnetic flux linking the circuit is changing. Consider the Fig. 2.1 shown alongside in which a magnetic field with flux density *B* is normal to the plane of the loop. If *B* is decreasing in magnitude and is directed upwards, then a current *I* flows in the wireloop as indicated in the figure. However, if *B* is directed upwards as before but is *increasing* in magnitude, the direction of induced current is opposite to that shown.

The direction of induced current is governed by the Right Hand Rule and Lenz's Law which states that "Induced current in the loop is always in such a direction as to produce flux opposing the change in *B*". The changing magnetic field produces an electric field, *E*. By integrating this field around the loop, an e.m.f., *e* results.

Faraday's Law gives a quantitative relation between the e.m.f. induced in a closed loop and the magnetic field producing the e.m.f. According to this law "The total e.m.f. induced in a closed circuit is equal to the time rate of decrease of total magnetic flux linking the circuit". Mathematically, we can write

$$e = -\frac{d\phi}{dt} \qquad ...(2.1)$$

where e = Total e.m.f. in volts

ϕ = Total flux in webers

t = Time in seconds.

Note that the negative sign indicates that the current direction and the e.m.f. direction is positive w.r.t., the direction of the field when the flux is *decreasing* with time. The above law Eq. (2.1) is valid for a single turn loop. For a loop with *N* turns, where all turns are linked by the same flux, ϕ, Faraday's law may be expressed as

$$e = -N\frac{d\phi}{dt} \qquad ...(2.2)$$

where N = number of turns.

Now the total flux through a circuit is equal to the integral of the normal component of the flux density, *B* over the surface bounded by the circuit. Hence we can write the total magnetic flux, ϕ is equal to

$$\phi = \int_s B.ds \qquad ...(2.3)$$

where the symbol $\int_s$ indicates integration over a surface *s*.

Substituting the value of ϕ from Eq. (2.3), into Eq. (2.1), we get

$$e = -\frac{d}{dt}\left\{\int_s B.ds\right\} \qquad ...(2.4)$$

where e = induced e.m.f. in volts

B = flux density in webers/m^2
ds = surface element in m^2
and t = time in seconds.

Eq. (2.4) may be applied to circuits in motion with respect to th observer, with flux density B either constant or time varyin provided that the motion of the boundary of the surface in Eq. (2.4) coincides at every instant with the motion of the parts of th physical circuit.

When the loop or closed circuit is stationary or fixed, Eq. (2.4) reduces to

$$e = -\int_s \frac{\partial B}{\partial t} . ds \qquad ...(2.5)$$

Also we know that

$$e = \oint E.dl \qquad ...(2.6)$$

where

E = the electric field produced by changing magnetic fields. From Eqs. (2.5) and (2.6), we have

$$e = \oint E.dl = -\int_s \frac{\partial B}{\partial t} .ds \qquad ...(2.7$$

where E = e.m.f. producing electric field in volts/m
e = induced e.m.f. in volts.
dl = element of path in metres
ds = element of area in (metres)^2
and B = flux density in webers/m^2.

Relation (2.7) is referred to as 'Maxwell's Equation as derive from Faraday's Law'. In Eq. (2.7) it appears in its integral form.

By means of Stokes Theorem*, if we substitute the surfac integral of the curl of E for the line integral of E in Eq. (2.7), we ge

$$\iint (\nabla \times E).ds = -\iint \frac{\partial B}{\partial t} .ds \qquad ...(2.$$

Since ds in Eq. (2.8) applies to any surface element, it i arbitrary and, therefore, the integrands of the above equation ar equal

i.e. $$\nabla \times E = -\frac{\partial B}{\partial t} \qquad ...(2.$$

---

* Refer to Appendix.

Eq. (2.9) is "*Maxwell's Equation* in differential form as derived from Faraday's Law".

Eq. (2.7) states "The line integral of electric field around a fixed closed loop or circuit is equal to the normal component of the time rate of decrease of flux density B integrated over a surface bounded by the circuit".

Eq. (2.9) states that "If B is changing with time at any point, then curl of E has a value different from zero at that point".

Eq. (2.9) as already stated is Maxwell's First Equation in Differential form as derived from Faraday's Law. Now we proceed to give a rigorous proof for the same as below.

We have to find the value of $\oint E.dl$.

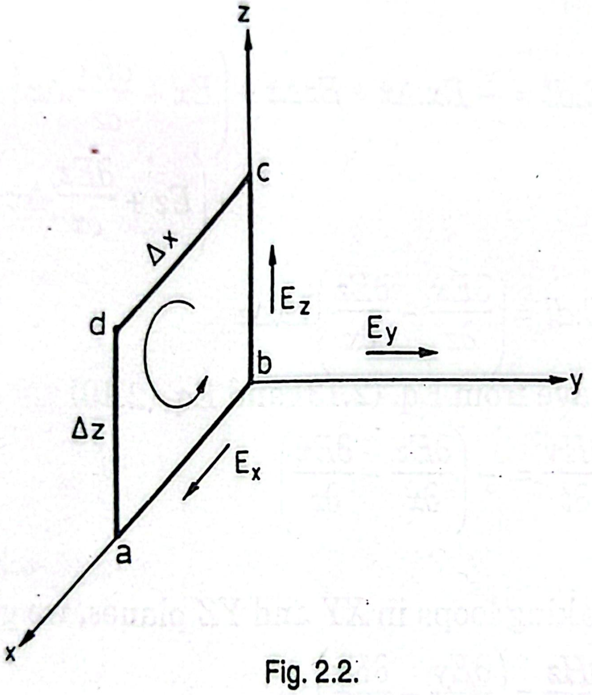


Fig. 2.2.

We have an arbitrary cuboid with dimensions Δx, Δy and Δz.

Considering a rectangular loop in XZ plane. The electric field along X-axis is Ex and that along Z-axis and Y-axis are Ez and Ey respectively.

We know $\oint E.dl = -\frac{\partial \phi y}{\partial t}$ for the loop in XZ plane

$$= -\frac{\partial By}{\partial t}(\Delta x\, \Delta z)$$

where $dA = \Delta x.\Delta z$,

$$\oint E.dl = -\mu \frac{\partial Hy}{\partial t}.\Delta x \Delta z \qquad \text{...(2.10)}$$

Here Hy = Magnetic field acting perpendicular to the plane of the wave.

Now $\oint_{abcd} E.dl = \int_{ab} E.dl + \int_{bc} E.dl + \int_{cd} E.dl + \int_{da} E.dl$ ...(2.1

Now $\overline{E}$ along $\overline{ab} = -Ex$

along $\overline{bc} = Ez$

along $\overline{cd} = Ex + \frac{\partial Ex}{\partial z} . \Delta z$ ...(2.1

along $\overline{da} = -\left(Ez + \frac{\partial Ez}{\partial x} . \Delta x\right)$

Substituting the various values from Eq. (2.12) in Eq. (2.11) et

$$\oint E.dl = -Ex.\Delta x + Ez\Delta z + \left(Ex + \frac{\partial Ex}{\partial z}.\Delta z\right)\Delta x - \left(Ez + \frac{\partial Ez}{\partial x}\Delta x\right)\Delta z$$

$$\oint E.dl = \left(\frac{\partial Ex}{\partial z} - \frac{\partial Ez}{\partial x}\right)\Delta x \Delta z$$ ...(2.1

hence we have from Eq. (2.13) and Eq. (2.10)

$$-\mu \frac{\partial Hy}{\partial t} = -\left(\frac{\partial Ez}{\partial x} - \frac{\partial Ex}{\partial z}\right)$$ ...(2.1

Similarly taking loops in *XY* and *YZ* planes, we get

$$-\mu \frac{\partial Hz}{\partial t} = \left(\frac{\partial Ey}{\partial x} - \frac{\partial Ex}{\partial y}\right)$$ ...(2.1

$$-\mu \frac{\partial Hx}{\partial t} = \left(\frac{\partial Ez}{\partial} - \frac{\partial Ey}{\partial z}\right)$$ ...(2.1

Now we have 3 equations *viz.* (2.14), (2.15) and (2.16). Multipl (2.16) by unit vector *i* ; (2.14) by *j* and (2.15) by *k*, where *i, j, k* ar unit vectors in the directions *X, Y, Z* respectively and adding th three equations, we get

$$-\mu\left[i\frac{\partial Hx}{\partial t} + j\frac{\partial Hy}{\partial t} + k\frac{\partial Hz}{\partial t}\right] = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ Ex & Ey & Ez \end{vmatrix}$$

$$\boxed{\Rightarrow -\mu\frac{\partial \overline{H}}{\partial t} = \nabla \times \overline{E} = -\frac{\partial B}{\partial t}} \qquad \text{...(2.17)}$$

Thus we see that Maxwell's first equation gives a relation between electric field and a time varying magnetic field.

2.2. Displacement Current

Now we introduce in this section a new concept of displacement current. Consider a RC combination in parallel as shown in Fig. 2.3. A constant voltage across the resistance, R produces a constant current of value I_1 such that

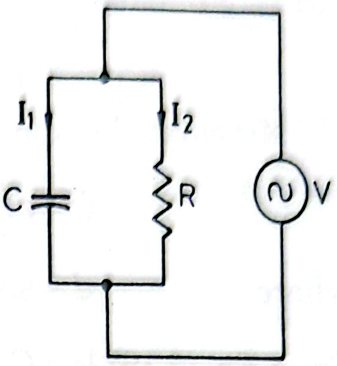


Fig. 2.3.

$$I_1 = \frac{V}{R} \qquad \text{...(2.18)}$$

whereas current through the capacitor C will be constant only when the voltage V is changing. That is current I_2 through the capacitor is given by

$$I_2 = \frac{dQ}{dt}$$

$$= \frac{CdV}{dt} \qquad \text{...(2.19)}$$

where the instantaneous charge Q on the capacitor is given by the relation

$$Q = CV \qquad \text{...(2.20)}$$

The current flowing through the resistor is known as *Conduction current,* while the current through the capacitor is commonly known as *Displacement current.* This is only a concept since the current actually does not flow through the capacitor. However, the external effect is as though it did, since as much current flows out of one plate as flows into the opposite one. The electric field inside each element equals the voltage V across the element divided by its length, d. That is

$$E = \frac{V}{d} \qquad \text{...(2.21)}$$

We know that the current density, J_1 inside the resistor is given by the relation

$$J_1 = \frac{I_1}{A}$$

$$= \sigma E \qquad \text{...(2.2}$$

where A = cross-sectional area

σ = conductivity of the medium of the resistor element.

Moreover the capacitance of a parallel plate capacitor is

$$C = \varepsilon \frac{A}{d} \qquad \text{...(2.23}$$

where d = spacing between the plates.

From Eq. (2.19), $I_2 = C \dfrac{dV}{dt}$

$$= \frac{\varepsilon A}{d} d \left(\frac{dE}{dt} \right)$$

or

$$I_2 = \varepsilon A \frac{dE}{dt} \qquad \text{...(2.24}$$

Therefore, the current density, J_2 inside the capacitor is given by

$$J_2 = \frac{I_2}{A} = \varepsilon \frac{dE}{dt} \qquad \text{...(2.25}$$

$$= \frac{dD}{dt} \qquad \text{...(2.25a)}$$

Since $D = \varepsilon E$

where D = Electric flux density.

It may be remembered that current density J ; electric field intensity E and electric flux density D, are space vectors having the same direction in an isotropic medium. We can express, in more general form, as

J_1 = conduction current density = $J_{cond.}$

$= \sigma E$

and J_2 = Displacement current density

$$= J_{dis.} = \varepsilon \frac{dE}{dt} = \frac{dD}{dt}$$

Hence the total current density, J_{total} is a sum of conduction current density and displacement current density.

i.e. $J_{total} = J_{cond} + J_{dis}$ ...(2.26

2.3. Maxwell's Second Equation

Ampere's law forms the basis for Maxwell's Second Equation. Ampere's Law states that 'Line integral of H around the closed loop contour is equal to the current enclosed'. The current referred to here is the sum of conduction and displacement currents, as the total current. Mathematically

$$\oint H.dl = \int_s (J_{cond} + J_{dis})\, ds \qquad \text{...(2.27)}$$

Substituting for J_{cond} and J_{dis} from Eqs. (2.22) and (2.25), we get

$$\oint H.dl = \int_s \left(\sigma E + \varepsilon \frac{\partial E}{\partial t}\right) . \, ds \qquad \text{...(2.28)}$$

Each term in the above equation has the dimensions of current and the integral is over the whole surface, Eq.(2.28) is the complete expression in integral form of *Maxwell's Equation Derived From Ampere's Law*. We can express it as

$$\oint H.dl = \int_s \left(J + \frac{\partial D}{\partial t}\right) . \, ds \qquad \text{...(2.29)}$$

where J refers only to conduction current density. By an application of Stoke's Theorem to Eq. (2.29) **Maxwell's Equation** in differential form results. That is

$$\nabla \times \overline{H} = \left(\overline{J} + \frac{\partial \overline{D}}{\partial t}\right) \qquad \text{...(2.30a)}$$

or

$$\nabla \times \overline{H} = \left(\sigma E + \varepsilon \frac{\partial E}{\partial t}\right) \qquad \text{...(2.30b)}$$

A vigourous proof of the above equation follows.

To prove equation (2.30) consider a loop in XZ plane having magnetic fields along X, Y, Z as Hx, Hy and Hz respectively.

$$\oint H.dl = I_i$$

For this loop

$$\left[\oint H.dl\right]_{xz} = I_{iy}$$

$$= J_{iy}\,(\Delta x \Delta z)$$

$$\oint H.dl = \left[J_{cy} + \frac{\partial Dy}{dt}\right] \Delta x \, \Delta z$$

$$\oint H.dl = \int_{ab} H.dl + \int_{bc} H.dl + \int_{cd} H.dl + \int_{da} H.dl$$

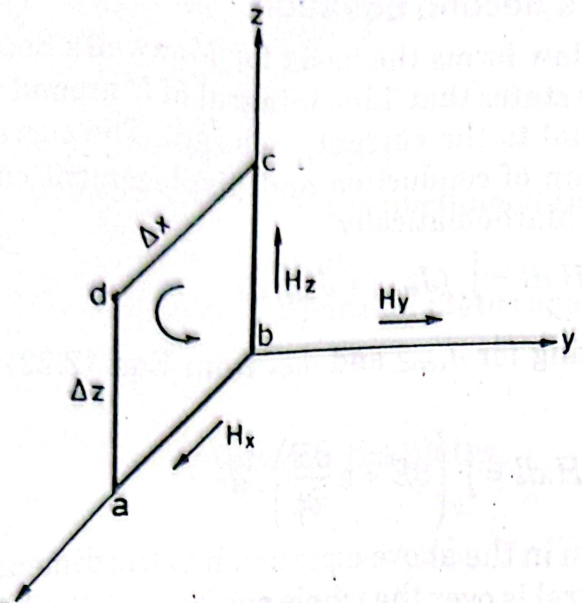


Fig 2.4.

$$\overline{H}_{ab} = -Hx\ ;\ \overline{H}_{bc} = Hz\ ;\ \overline{H}_{cd} = Hx + \frac{\partial Hx}{\partial z}\Delta z$$

and $$\overline{H}_{da} = -\left(Hz + \frac{\partial Hz}{\partial x}\Delta x\right)$$

So that

$$\oint_{abcd} H.dl = (-Hx)\,(\Delta x) + Hz\Delta z + \left(Hx + \frac{\partial Hx}{\partial z}\Delta z\right)\Delta x + (-\Delta z)\left(Hz + \frac{\partial Hz}{\partial x}\Delta x\right)$$

$$\oint_{abcd} H.dl = -\left(\frac{\partial Hz}{\partial x} - \frac{\partial Hx}{\partial z}\right)\Delta x\Delta z \qquad \text{...(2.31)}$$

$$= \left(J_{cy} + \frac{\partial Dy}{\partial t}\right)\Delta x\ \Delta z$$

Similarly for loops perpendicular to *X* and *Z*, we have

$$\left[J_{cx} + \frac{\partial Dx}{\partial t}\right] = -\left[\frac{\partial Hy}{\partial z} - \frac{\partial Hz}{\partial y}\right] \qquad \text{...(2.32)}$$

and $$\left[J_{cz} + \frac{\partial Dz}{\partial t}\right] = \left[\frac{\partial Hy}{\partial x} - \frac{\partial Hx}{\partial y}\right] \qquad \text{...(2.33)}$$

Multiplying Eqs. (2.31), (2.32) and (2.33) by *j*, *i* and *k* respectively and adding then we have

$$\nabla \times H = J_x i + J_y i + J_z k = \overline{J}$$

or
$$\nabla \times H = \left[\bar{J}_c + \frac{\partial \bar{D}}{\partial t}\right]$$

and
$$\int (\nabla \times H).ds = I + \int \frac{\partial D}{\partial t} . ds = \oint H.dl \qquad ...(2.34)$$

and
$$\nabla \times H = J = J_c + J_d$$
$$= \left[\sigma E + \frac{\partial D}{\partial t}\right] \qquad ...(2.35)$$

Maxwell's Third Equation. We know that the total electric flux over the sphere (obtained by integrating the normal component of electric flux density D over the sphere) is equal to the charge Q enclosed by the sphere. A more general statement of Gauss law for a special case is "the surface integral of the normal component of the electric flux density D over any closed surface equals the charge enclosed". Symbolically we can write

$$\iint D.ds = Q \qquad ...(2.36)$$

where Q is the total charge enclosed. Note that this statement of "Gauss law" is applicable specifically to the rationalised MKS system. In general the law may be redefined as "the surface integral of the normal component of electric flux density over a closed surface is proportional to the charge enclosed (or is equal to the charge multiplied by a constant, this constant assuming the value unity in the retionalised MKS system)".

This charge Q can also be expressed as the volume integral of the charge density ρ so that Eq. (2.36) may be rewritten as

$$\iint_s D.ds = \iiint_v \rho.dv = Q \qquad ...(2.37)$$

Here the surface integration is carried out over a closed surface and the volume integration throughout the region enclosed.

If a volume is devoid of any charge then the electric flux over the surface of the volume is always zero even though the volume may be in an electric field. In such a case the net flux is zero since the inward flux is equal to the outward flux.

We know by the definition of divergence that

$$\nabla.D = \frac{\partial Dx}{\partial x} + \frac{\partial Dy}{\partial y} + \frac{\partial Dz}{\partial z} \qquad ...(2.38)$$

So that

$$\int_v \nabla.D\, dV = \iiint \left(\frac{\partial Dx}{\partial x} + \frac{\partial Dy}{\partial y} + \frac{\partial Dz}{\partial z}\right) dx\, dy\, dz \qquad ...(2.39)$$

where $dV = dx\, dy\, dz$,

dV is an elemental volume.

Therefore, $\int_v \nabla.D\, dV = \oint_s D.ds$ ...(2.40)

Now from Eq. (2.37), we know that

$$\iint D.ds = \iiint \rho.dV = Q$$

Applying Eq. (2.40) to the above equation, we get

$$\int_v \nabla.D.dv = \int_v \rho.dV$$

Since the volume chosen is arbitrary, the above relation holds for any volume whatsoever. Moreover as the volume considered is reducible to an elemental volume, this becomes a point relation and the integrands can be equalised, *i.e.*

$$\nabla.D = \rho \qquad ...(2.41)$$

This is an alternative statement of Gausses law and states that "At every point in a medium the divergence of electric displacement density is equal to the charge density".

The physical interpretation of Eq. (2.41) may be stated as follows :

The net outward flux of electric displacement per unit volume is equal to the charge per unit volume. Eq. (2.41) is a more useful form for mathematical manipulation then the corresponding integral statement given in Eq. (2.37).

Maxwell's Fourth Equation. The magnetic flux density B at any point P produced by a current distribution is given by

$$B = \frac{\mu}{4\pi} \iiint \frac{J \times a_\gamma}{\gamma^2} . dv \qquad ...(2.42)$$

where B is flux density in weber's per metre square

μ = permeability of medium considered to be uniform.

J = Current density at volume element in ampere's per m^2.

a_γ = Unit vector in direction of radius vector γ.

dv = Volume element.

The construction of current distribution is shown in Fig. 2.5.

By carrying out the triple integration as indicated, the total flux density B at P due to current is obtained. Thus we can write B as a curl of some other function A known as vector potential, *i.e.*

$$B = \text{curl of } A = \nabla \times A.$$

Moreover the divergence of the curl of a vector function is zero, *i.e.* divergence of curl of A is equal to divergence of B is equal to zero.

Mathematically :

$$\nabla \,.\, B = 0 \qquad ...(2.43)$$

This is known as Maxwell's fourth equation. The corresponding equation in integral form is

$$\oint B \,.\, ds = 0.$$

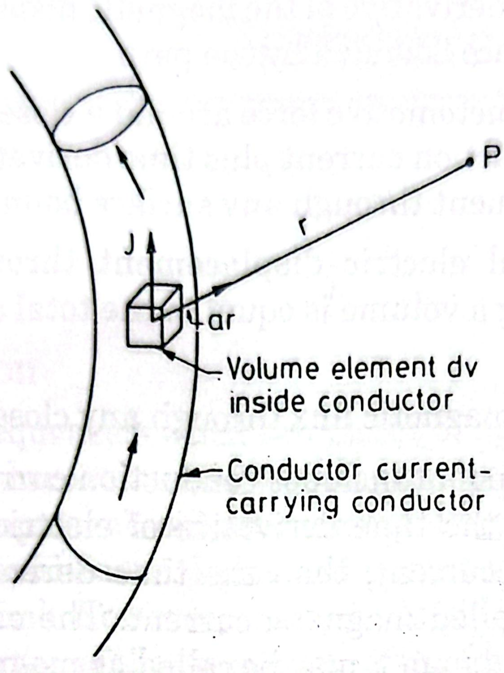

P
r
J
ar
Volume element dv inside conductor
Conductor current-carrying conductor

Fig. 2.5 Current distribution.

The proof is left to the reader as an exercise.

[Hint. Apply Stokes theorem.]

Word statement of Maxwell equations. Equations (2.17), (2.30), (2.41) and (2.43) are known as Maxwell equations since Maxwell was responsible for their development and establishment as self-consistent set. Each differential equation has its integral counter part and the two forms can be derived from each other by application of Divergence theorem or Stokes Theorem (Refer Appendix). We can repeat the above equations as follows.

$$\nabla \times E = \frac{\partial B}{\partial t}, \oint E.dl = -\int_s \frac{\partial B}{\partial t} \,.\, ds \qquad ...(i)$$

$$\nabla \times H = \left(\sigma E + \varepsilon \frac{\partial E}{\partial t}\right),$$

$$\oint H.dl = \int_s \left(\sigma E + \varepsilon \frac{\partial E}{\partial t}\right).ds \qquad ...(ii)$$

$$\nabla . D = \rho, \iint D . ds = \iiint \rho . dv \qquad ...(iii)$$

$$\nabla . B = 0, \int B . ds = 0 \qquad ...(iv)$$

A word statement of the significance of the field equations is readily obtained from their mathematical statement in the integral form. It would be as follows.

(*i*) The electromotive force around a closed path is equal to the time derivative of the magnetic displacement through any surface bounded by the path.

(*ii*) The magnetomotive force around a closed path is equal to the conduction current plus time derivative of the electric displacement through any surface bounded by the path.

(*iii*) The total electric displacement through any surface enclosing a volume is equal to the total charge within the volume.

(*iv*) The net magnetic flux through any closed surface is zero.

The electric current includes conduction current and displacement currents. If the time derivative of electric displacement is called an electric current, than the time derivative of magnetic displacement is called magnetic current. The e.m.f. is known as electric voltage and m.m.f. may be called as magnetic voltage.